Lecture 14: Weak Interactions (I): Beta decay, Neutrinos and Parity Violation

October 11, 2016

Outline

- Nuclear β-decay
- Four Fermi Interactions
- Inverse β-decay
- From four-Fermi Theory to Intermediate Vector Bosons
- Parity Violation ($V - A$) (to be continued Thursday)

Nuclear β-Decay

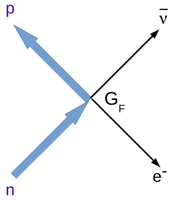
- First observed weak decay

$$n \to p + e^- + \overline{\nu}_e$$

- Existence of ν first proposed by Pauli:
  - Trajectory of e^- not co-linear with recoiling nucleus and no additional particle seen
    Conservation of momentum $\to$ additional decay product (the ν)
  - Electron does not have a discrete energy (3-body decay)
  - Endpoint of e energy spectrum close to maximum allowed for 2-body decay: $m_\nu \sim 0$
  - Change in nuclear spin is 0 or ± 1 *never* ± 2
    - Since e has spin-$\frac{1}{2}$, angular momentum conservation tells us the ν has spin-$\frac{1}{2}$

Four-Fermi Interaction

- Fermi assumed weak decay occurs via hadronic weak current $\langle p | J_\mu^{wk} | n \rangle$ and leptonic weak current $\langle e\nu | J_\mu^{wk} | 0 \rangle$



- The complete matrix element was written

$$M_{if} = \langle p | J_\mu^{wk} | n \rangle \langle e\nu | J_\mu^{wk} | 0 \rangle$$

- Current-current form implies existence of purely leptonic processes, eg

$$\mu^- \to e^- + \overline{\nu_e} + \nu_\mu$$

and purely hadronic weak processes, eg

$$\Lambda \to p\pi^-$$

- Strength of interaction set by a constant G_F, assumed to be universal

Decay Rates and Fermi's Golden Rule

- Transition rate W:
$$W_{fi} = 2\pi G_F^2 |M_{if}|^2 \mathcal{D}(E_f)$$
where $G_F = 1.16637 \times 10^{-5}$ GeV^{-2} and $\mathcal{D}$ is the density of states
  - Note: the fact that G_F is not a dimensionless coupling constant tells us that something is going on. We'll talk about this in a few minutes
- The density of states
$$d^2N = p_e^2 dp_e \; p_\nu^2 dp_\nu$$
For a massless neutrino (and ignoring small nuclear recoil)
$$p_\nu = (E_f - E_e); \quad dp_\nu = dE_f$$
Thus
$$\frac{dN}{dE_f} = p_e^2 (E_f - E_e)^2 dp_e$$
- Assume for now that $|M|^2$ is constant. So, the electron spectrum is
$$N(p_e) dp_e \propto p_e^2 (E_f - E_e)^2 dp_e$$
- Modification for non-zero neutrino mass
$$N(p_e) \propto p_e^2 (E_f - E_e)^2 \left[1 - \frac{m_\nu}{(E_f - E_e)}\right] dp_e$$

The Kurie Plot and ν Mass

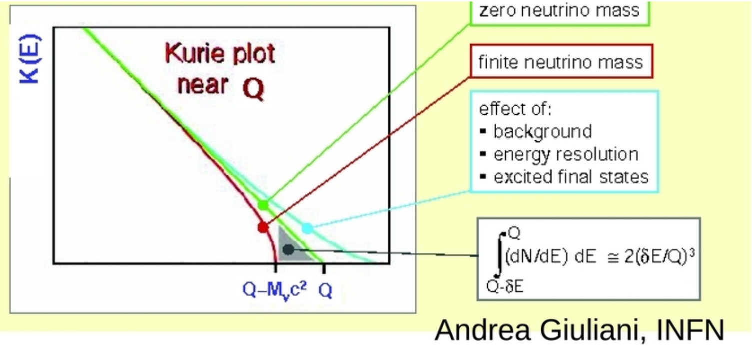
- From previous page:

$$N(p_e)dp_e \propto p_e^2(E_f - E_e)^2 \left[1 - \frac{m_\nu}{(E_f - E_e)}\right] dp_e$$

Thus

$$\sqrt{N(p_e)/p_e^2} \propto (E_f - E_e) \sqrt{1 - \frac{m_\nu}{E_f - E_e}}$$

- This is called a Kurie plot



Issues in Direct Measurement of ν Mass

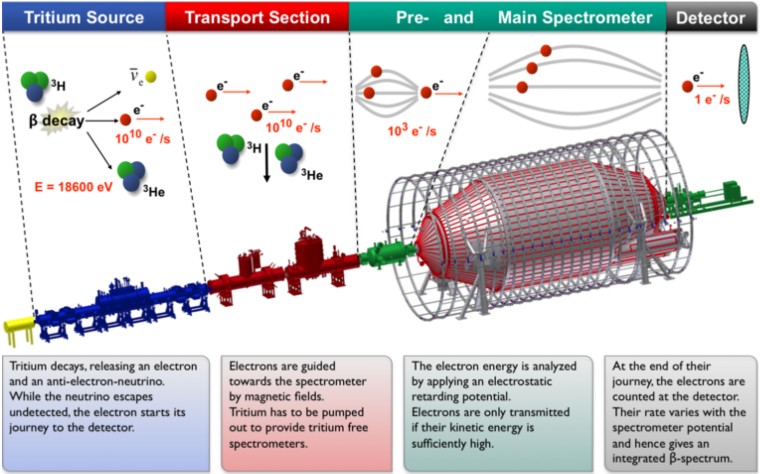
- Counting rate near endpoint is only a small fraction of total decay rate
- Integrating the β-spectra over interval ΔE from the endpoint, rate

$$R \propto (1 - [m_\nu/\Delta E]^2)^{3/2}$$

  Assuming a spectrometer resolution of 10 eV, number of events has to be increased by factor of ~ 15 to improve sensitivity from $m_\nu = 10$ eV to $m_\nu = 5$ eV
- The thickness of the source must be accounted for very accurately: energy loss of the electrons for a dense source
- Binding energy corrections for the nuclei can be important: This is why many modern experiments go to Tritium

PDG Limit: $m_{\nu_e} < 2$ eV (90% CL)

The Next Generation of Direct Mass Measurement: Katrin



- β-decay from Tritium gas
- Large volume for high rate
- Low temperature to (30K) to reduce thermal motion

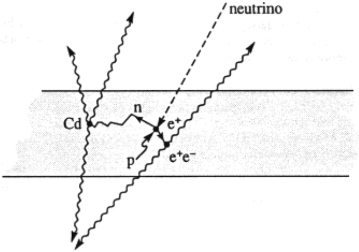
Inverse β-Decay

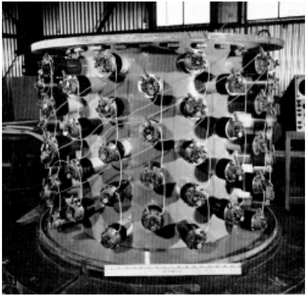
- Pauli and Fermi's explanation of β-decay postulated the existence of the ν, but it wasn't until 1959 that the particle nature was observed
- Inverse β-decay

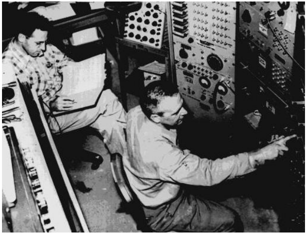
$$\overline{\nu}_e + p \to n + e^+$$

- Reines and Cowan (Phys Rev 113 91959) 272) use Savannah River reactor as a source of $\overline{\nu}$ and a $CdCl_2 + H_2O$ target/detector
  - e^+ comes to rest (ionization energy loss) and forms positronium
  - Positronium decay to 2γ which produce electrons by Compton effect $(10^{-9}$ sec)
  - Cd captures the neutron after it has moderated in H_2O. Radiative γ rays from neutron capture (μsec time scale)
- Signal consisted of 2 pulses separated in time by a few μsec.
- Rate can be estimated assuming matrix element related to β-decay
- Observations:
  - The $\overline{\nu}_e$ is a real massless or nearly massless particle
  - Rate consistent with predictions from Fermi theory

Pictures of the Reines and Cowan Experiment







From Four-Fermion Coupling to Intermediate Vector Bosons

- Why does G_F has dimensions of GeV^{-2}?
  - Four-fermion coupling does not include a $1/q^2$ propagator
- Replace 4-point interaction with the exchange of $W^{\pm}$ boson with mass M_W
  - For QED, the propagator is
$$e\frac{-g^{\mu\nu}}{q^2}$$
  - With massive intermediate boson we get
$$g_{wk}\frac{-g^{\mu\nu}+q^{\mu}q^{\nu}/M_W^2}{q^2-M_W^2}$$
  - Matrix element becomes
$$M \sim g_{wk}j_{\mu}^{wk}(\frac{-g^{\mu\nu}+q^{\mu}q^{\nu}/M_W^2}{q^2-M_W^2})j_{\nu}^{wk}$$
  - For small q^2, we get
$$M \sim g_{wk}(q^2 \to 0)j_{\mu}^{wk}(\frac{g^{\mu\nu}}{M_W^2})j_{\nu}^{wk}$$
  - Can identify
$$G_F = \frac{g_{wk}}{M_W^2}$$
    thus large M_W means small G_F

A Unitary Argument for the Vector Boson Theory (I)

- Suppose the four-fermion theory were right:
  - Using dimensional analysis: $e\nu$ scattering cross section

$$\sigma(e\nu \to e\nu) \propto G_F^2 s$$

  - Similar expression for νp scattering, except with convolution over pdf's
  - Low energy ν scattering data agrees with this result
- If this formula holds to all energies, we have a problem
  - No cross section can exceed the unitarity bound
  - Write as a sum over partial waves

$$\sigma_{TOT} = \frac{4\pi}{k^2} \sum_J (2J+1)|f_J|^2$$

    where k is the cm momentum
  - Flux conservation $\to |f_J| \leq 1$
  - The cross section in each partial wave is bounded

$$\sigma_J \leq \frac{2\pi(2J+1)}{k^2} \Rightarrow \sim \frac{1}{s}$$

    as s increases, the bound falls
  - At $\sqrt{s} \sim 500$ GeV, unitary it violated

A Unitary Argument for the Vector Boson Theory (II)

- This argument told physicists that four point theory would fail at high energies and argued for the intermediate boson theory
- Note: We can estimate m_W if we assume $g_{wk} \approx e$:

$$G_F \sim g_{wk}^2/M_W^2 \Rightarrow M_W \sim e/\sqrt{G_F} \approx 100 \text{ GeV}$$

The W was first observed in 1982
We'll come back to that part of the line story later

NB: Very similar arguments were used to demonstrate that EWSB must have measurable effects on the TeV scale

- Helped justify choice of LHC energy

Summary of What We Have Learned So Far

- QED is a remarkably successful theory that describes EM interaction of charged leptons and photons
- Neutral leptons (neutrinos) also exist and are produced in β-decay
- β-decay is not a QED process. Fermi described it with a 4-fermion interaction. This describes:
  - The β-decay spectrum
  - Inverse β-decay
  - Existence of both purely leptonic and purely hadronic weak decays
- In analogy with QED, we can replace this interaction with exchange of a massive charged vector boson, the W:
  - Avoids unitarity crisis
  - Explains why weak interactions are weak
  - If $g = e$, $M_W \sim 100$ GeV

In 1956, a MAJOR change in the model:
Observation of Parity Violation

Review: Parity

- Parity operator defined as spatial inversion

$$(x, y, z) \longrightarrow (-x, -y, -z)$$
$$P(\psi(\vec{r}) = \psi(-\vec{r})$$

- Parity conserved in strong and EM interactions
- Can classify parity of different operators:

Name	Form	Parity	Example
Scalar	$\overline{\psi}\phi$	$+1$	Temperature
Pseudoscalar	$\overline{\psi}\gamma^5\phi$	-1	Helicity
Vector	$\overline{\psi}\gamma^\mu\phi$	-1	Momentum
Axial Vector	$\overline{\psi}\gamma^\mu\gamma^5\phi$	$+1$	Angular Momentum
Tensor	$\overline{\psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$	$+1$	$F^{\mu\nu}$

The θ-τ Puzzle

- In 1950's, bubble chamber measurements resulted discovery of many hadrons
- Among them, the (then called) θ^+ and τ^+ (Warning: this has nothing to do with the τ lepton)
- Properties of θ and τ:
  - Strong production
  - Same mass: 493 MeV
  - Same Lifetime: 1.2×10^{-8} sec: weak decay (strange particles)
  - Spin 0
  - Different decay modes:

$$
\begin{aligned}
\theta^+ &\to \pi^+\pi^0 & \mathrm{P} = +1 \\
\tau^+ &\to \pi^+\pi^+\pi^- & \mathrm{P} = -1
\end{aligned}
$$

- If P conserved, these must be different particles

Why do they have the same mass and lifetime?

An Aside: How do we know the parity of the final states?

- $\theta^+ \to \pi^+\pi^0$
  - Spin 0 particle decays to two spin 0 particles:
  $$\ell = 0$$
  - Parity from angular momentum and intrinsic parity:
  $$P = (-1)^\ell(-1)^2 = 0$$
- $\tau^+ \to \pi^+\pi^+\pi^-$
  - $\pi^+\pi^+$ must have even ℓ (Bose Statistics)
  - If $\ell(\pi^+\pi^+) = 0$, angular momentum of π^- wrt this system also 0 and
  $$P = (-1)^3$$
  - If $\ell(\pi^+\pi^+) = 2$, more possibilities

You will learn more about this on Homework # 7

Lee and Yang's Suggestion

- At 1956 Rochester meeting, question raised whether θ and τ could be the same particle
- Lee and Yang did extensive analysis of existing tests of P conservation. Conclusion:
  - Stringent tests of P conservation for strong and EM interactions
  - No evidence for P conservation in weak decays
- Suggested tests of P conservation in weak decays:
  - Look for interactions that differentiate that left and right handed amplitudes
  - Since decay rate $\propto |\mathcal{M}|^2$, must look for interference between amplitudes of opposite parity
  - Express decay rate as sum of scalar and pseudoscalar terms
  - Identify possible pseudoscalars constructed from observables in decay of particle $P \to P_1 + P_2 + P_2$:
    - $\vec{p}_1 \cdot (\vec{p}_2 \times \vec{p}_3)$
    - $\vec{p}_1 \cdot \vec{S}$ (if P_1 has spin $\vec{S}$)

CS Wu's Discovery of Parity Violation (I)

- Look at relative β-decay rate $||$ and anti-$||$ to direction of polarization for a polarized nucleus
- Worked with Co^{60} $(J^P = 5^+)$ (half-life: 5 years)
- Decay product: Ni^{60} $(J^P = 4^+)$
- Change of angular momentum without change in parity
  - e and ν_e must have $J = 1$
- To polarize Co^{60} need B field and low temperature
- Cool to $0.1°$K
- Need state-of-the art (for then) refrigeration
- Experiment done at Bureau of Standards in Maryland

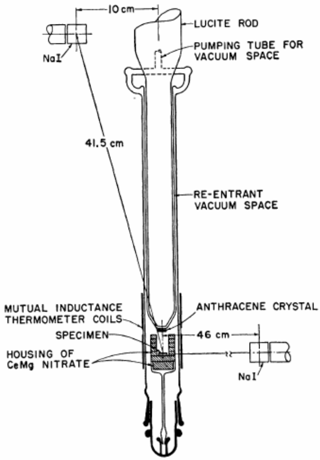


FIG. 1. Schematic drawing of the lower part of the cryostat.

CS Wu's Discovery of Parity Violation (II)

- Monitor level of polarization by studying photons produced in Ni decay
  - Two NaI crystals in polar and equitorial plane used to measure anisotropy
- Co^{60} source allowed to warm up: polarization disappears
- Also can change sign of B field
- Result shows β intensity

$$I(\theta) = 1 + \alpha \frac{\sigma \cdot p}{E}$$

with α negative

- Can't measure α but it is large (consistent with -1)
- Later work by Fraenfelder: $\alpha = -1$

The ν has a single handedness!

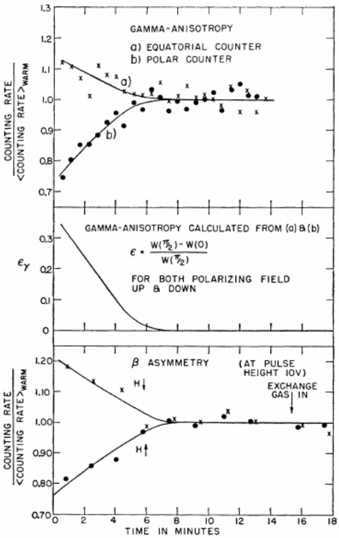
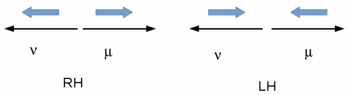


FIG. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.

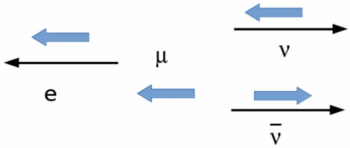
Garwin and Lederman: Confirmation of Parity Violation (I)

- Results of Wu et al led to flood of experiments
- First appeared in same Phys Rev issue as Wu
- Study $\pi^+ \to \mu^+\nu$, $\mu^+ \to e^+\nu_e\overline{\nu}_\mu$
- Since π^+ has spin 0, μ and ν must have $S = 0$



- If parity not conserved two possibilities need not be present equally
- Thus, μ will be polarized

- When μ decays, polarization results in asymmetry in direction of emission of electron (since ν has a single handedness)
- For the case where ν is left-handed:



If ν were right-handed, just reverse ν and $\overline{\nu}$ labels

- In either case, electron will exhibit an asymmetry

$$I(\cos\theta) = 1 + \alpha\cos\theta$$

Garwin and Lederman: Confirmation of Parity Violation (II)

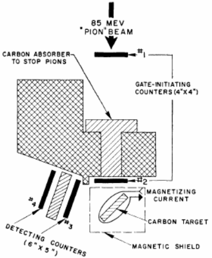


FIG. 1. Experimental arrangement. The magnetizing coil was close wound directly on the carbon to provide a uniform vertical field of 79 gauss per ampere.

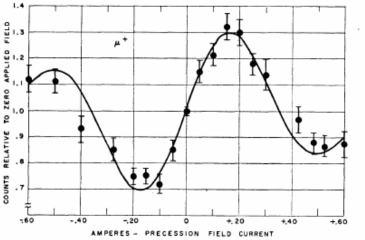


FIG. 2. Variation of gated 3–4 counting rate with magnetizing current. The solid curve is computed from an assumed electron angular distribution $1-\frac{1}{3}\cos\theta$, with counter and gate-width resolution folded in.

- Apply small vertical B field to allow μ to precess
- Rate at fixed angle depends on precession speed and on polarization
- Possible to map complete distribution with one fixed counter

- First measurement of g for the μ: $g = 2$ as expected
- Clear evidence for parity violation
- Repeated experiment with π^- and saw asym change sign

The Helicity of the Neutrino (Goldhaber et al)

- Begin with Eu^{132} (spin 0)
- Allow e^- capture to get $Sm^{152\,*}$ $(J = 1)$
- Spin of Sm^* always in same direction as e^-
- $Sm^{152\,*} \rightarrow Sm + \gamma$ (Sm has $J = 0$)
  $\Rightarrow \gamma$ has helicity of Sm^* in forward direction
- Select forward γ: Use Sm target. Forward γ has enough energy to interact. Backward doesn't "Resonant scattering"

$$\gamma + Sm^{152} \rightarrow Sm^{152\,*} \rightarrow \gamma + Sm^{152}$$

- Measure polarization by passing γ through magnetized iron
  electron with spin opposite that of photon can be absorbed
- If γ beam in same direction as B, transmission is greater for left-handed than for right-handed γ's

The ν is left-handed!

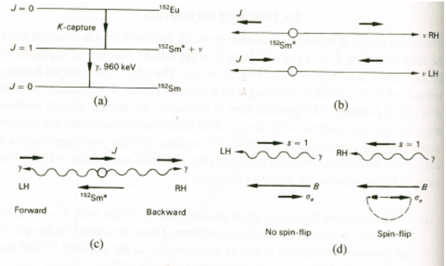


Fig. 7.7. Principal steps in the experiment to determine the neutrino helicity, as described in the text.

- From Perkins, Introduction to High Energy Physics
- See also Goldhaber and Cahn for discussion of this experiment

Incorporating V-A into Fermi's 4-Point Interaction

- For EM, $J = \overline{\psi}\gamma_\mu\psi$ where γ_μ is a vector operator
- For WI, will generalize to $J = \overline{\psi}\mathcal{O}\psi$
- What possible Lorentz forms can $\mathcal{O}$ have?
  - S,P Spin 0: ℓ and $\overline{\ell}$ have same helicity
  - V,A Spin 1: ℓ and $\overline{\ell}$ have opposite helicity
  - T Spin 2: ℓ and $\overline{\ell}$ have opposite helicity
- Experiments have shown that only V and A currents exist
- Note: $\mathcal{O}$ for leptons and for quarks doesn't have to be the same (we need to check!)
  - Also, hadrons have SI corrections that can modify the ratio of V to A
- For leptons:

$$J_{lept} = \psi_e\gamma_\mu(\alpha + \beta\frac{\sigma \cdot p}{E})\psi_\nu$$

  if $\beta = -\alpha$, LH ν.
- Experimentally, for leptons:

$$\begin{aligned} J_{lept} &= \psi_e\gamma_\mu\frac{1}{2}\left(1 - \frac{\sigma \cdot p}{E}\right)\psi_\nu \\ &= \psi_e\gamma_\mu\frac{1}{2}\left(1 - \gamma_5\right)\psi_\nu \end{aligned}$$

Helicity and Chirality

- For massless fermions, operator to project states of particular helicity are:

$$\begin{aligned} P_R &= \frac{1}{2}\left(1 + \frac{\sigma \cdot \mathbf{p}}{E}\right) \\ P_L &= \frac{1}{2}\left(1 - \frac{\sigma \cdot \mathbf{p}}{E}\right) \end{aligned}$$

- For massive fermions, need 4-component spinor and 4-component operator

$$P_{L,R} = \frac{1}{2}\left(1 \pm \gamma_5\right)$$

- Because direction of spin wrt momentum changes under boosts, this operator cannot represent helicity per se
- Instead, projects out state of polarization $P = \pm v/c$
  - In spite of this, everyone writes

$$\frac{1}{2}\left(1 - \gamma^5\right)\right) u \equiv u_L$$

$\frac{1}{2}\left(1 \pm \gamma^5\right)$ are called the chiral projection operators

Classification of Weak Decays

- Leptonic: only leptons in final state. Eg:

$$\begin{aligned} \mu^- &\to e^-\overline{\nu}_e\nu_\mu \\ \pi^- &\to \mu^-\overline{\nu}_\mu \end{aligned}$$

- Semileptonic: Both leptons and hadrons in final state. Eg:

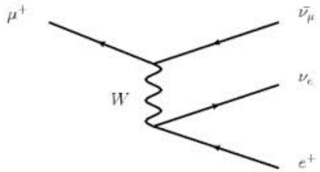
$$\begin{aligned} n &\to pe^-\overline{\nu}_e \\ K^0 &\to \pi^0e^+\nu_e \end{aligned}$$

- Hadronic: Only hadrons in final state. Eg:

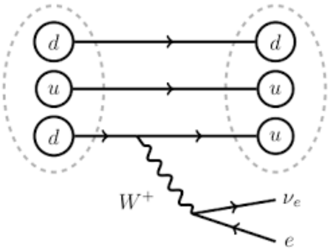
$$\begin{aligned} K^0 &\to \pi^+\pi^- \\ \Lambda &\to p\pi^- \end{aligned}$$

Example Feynman Diagrams for Weak Decays

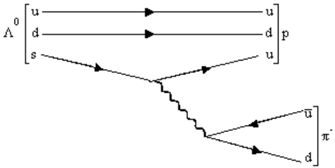
μ-decay



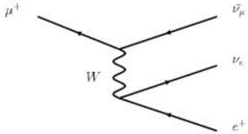
β-decay



Λ-decay



Muon Decay



- From dimensional analysis that $\Gamma \propto G_F^2 m_\mu^5$
  - Implicitly assumes couplings to e and μ are same

$$G_F^e = G_F^\mu$$

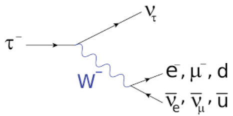
- Full calculation gives

$$\Gamma_\mu \equiv \frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^2}$$

where $G_F = g_{wk}/{m_W}^2$

  - Full spinor calculation can be found in many books, including Griffiths *Introduction to Elementary Particles*

Tau Decay



- $m_\tau = 1.777$ GeV
- Several possible decays:

$$\begin{aligned} \tau^- &\rightarrow e^-\overline{\nu}_e\nu_\tau \\ \tau^- &\rightarrow \mu^-\overline{\nu}_\mu\nu_\tau \\ \tau^- &\rightarrow d\overline{u}\ \nu_\tau \end{aligned}$$

In last case, the $d\overline{u}$ turns into hadrons with 100% probability

- All diagrams look like μ-decay
- If $G_F^\mu = G_F^e = G_F$, predict:

$$\begin{aligned} \Gamma_{\tau^-\rightarrow e^-} &= \Gamma_{\tau^-\rightarrow\mu^-} \\ &= (m_\tau/m_\mu)^2\,\Gamma(\mu) \end{aligned}$$

(difference in available phase space)

- Using the measured τ-lifetime and BR, check consistency of G_F

$$\begin{aligned} G_F^\tau/G_F^\mu &= 1.0023 \pm 0.0033 \\ G_F^e/G_F^\mu &= 1.000 \pm 0.004 \end{aligned}$$

Lepton universality for G_F

- For quark decays, need a factor of 3 for color. Predict

$$BR(\tau \rightarrow hadrons) = \frac{3}{3+1+1} = 60\%$$

- Experimental result:

$$BR(\tau \rightarrow hadrons) = (64.76 \pm 0.06)\%$$

Difference from 60% understood (QCD corrections; as for R)

Next time: G_F for quarks